Pruning Neural Networks with Matrix Methods

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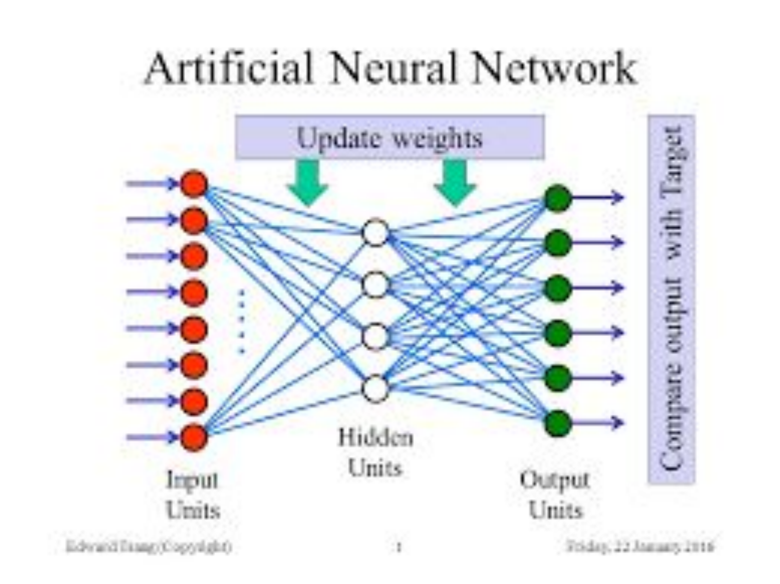
Background

- Neural Networks are the state-of-the-art models for tasks such as *classification*, *regression*, *generative modeling*, etc



- Most networks are very large
  - AlexNet (240Mb, 21 M parameters)
  - ChatGPT (500Gb, 175 B parameters)

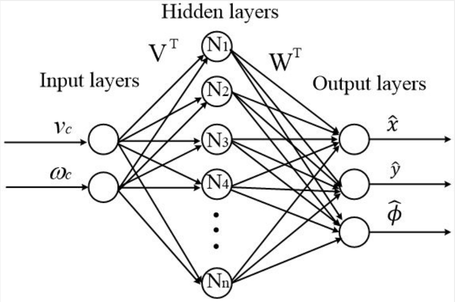
- Cons: High Inference Time, Need lots of compute, Limited applications in edge computing

- Pruning aims to reduce redundant parameters

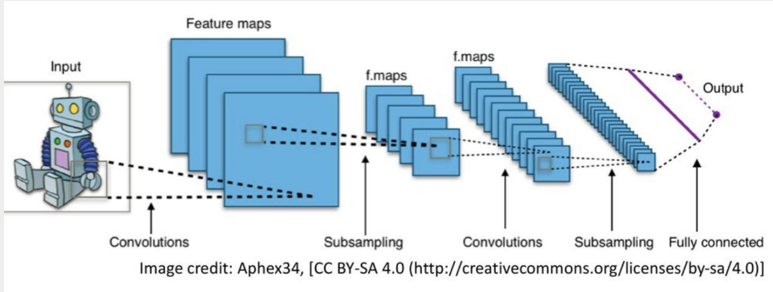
Methods

Fast Post-training Pruning

- Linear layer == Matrix Multiplication,
- Convolutional Layer ~ Matrix Multiplication
    - Not quite the same but it is a linear operation
- We can approximate each operation with techniques from this class
    - **PCA** for dimension reduction
    - **Low-rank approximation** for fast multiplication
    - **Randomized linear algebra** for fast multiplication



Linear Layer



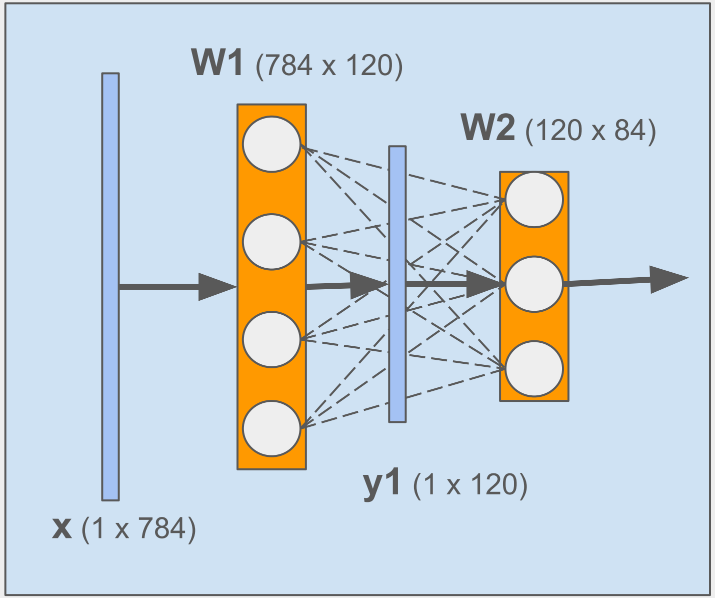
Convolutional Layer

PCA For Dimension Reduction

- **Goal**: Reduce the dimensionality of intermediate features

- **Expected Result:**
  - Applicable for taking high dimensional input to low dimensional output
  - Ex. Image Encoders, Image/Text Classification
  - Reduce the Size & Inference Time of the network

- **Procedure**:
  - PCA to reproject **W1** to a smaller output feature space
  - Preserve most of the variance between the rows of **W1**
  - Projection matrix: PCA(**W1**) => **P** (120 x 80)
  - Reproject
    - **W1* = W1 * P** (784 x 80)
    - **W2* = $\mathbf{P^T}$ * W2** (80 x 74)
  - Essentially, we have reprojected **y1* = y1 * P**



Neural Network

*Input: **x***

*Layer 1 output: **y1***

*Layer 1 Weight: **W1***

*Layer 2 Weight: **W2***

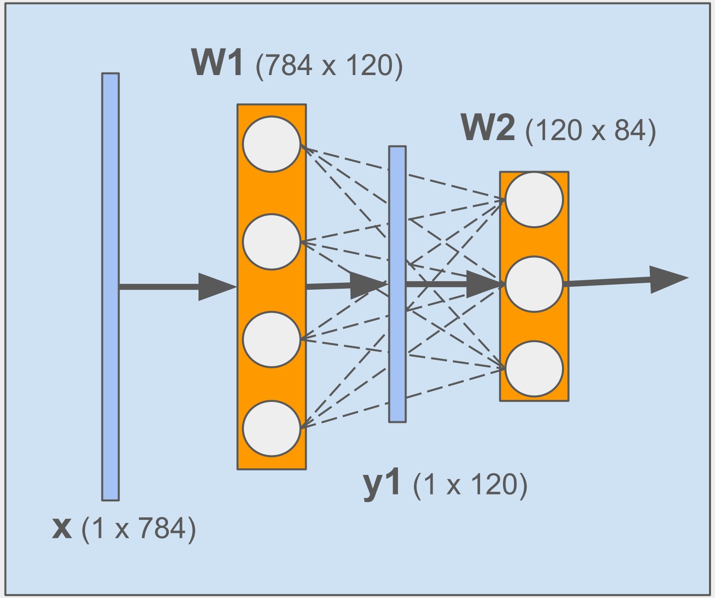
y1 = x * W1

Low-Rank Approximation

- **Goal**: Make Matrix Multiplication Faster

- **Expected Result:**
    - Inference time should be drastically reduced for networks with many redundant parameters

- **Procedure**:
    - Use SVD to compute a low rank approximation of **W1**
    - **W1 = U * V** where **U** (784 x k) & **V** (k x 120), *k << 120*
    - **y1 = x * W1 ===> y1 = x * U * V**
    - Computation time: 784 * 120 ===> 884 * k



Neural Network

Input: ***x***

Layer 1 output: ***y1***

Layer 1 Weight: ***W1***

Layer 2 Weight: ***W2***

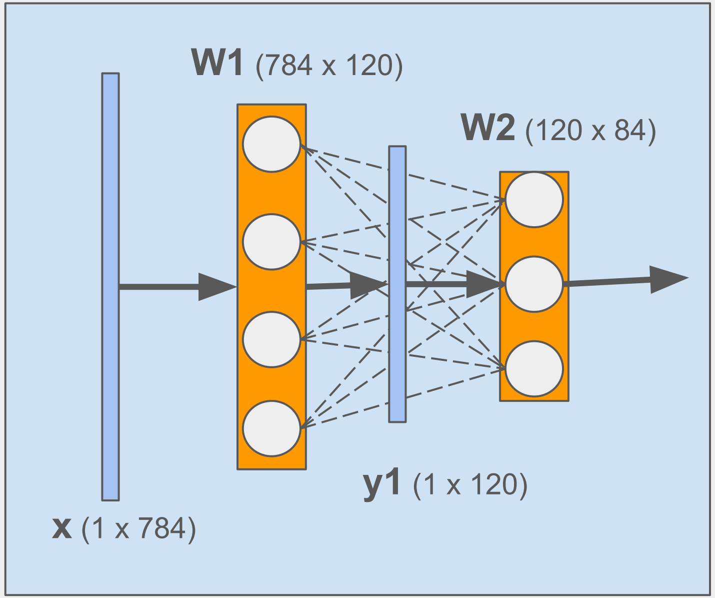
y1 = x * W1

Randomized Linear Algebra

- **Goal**: Make Matrix Multiplication Faster

- **Expected Result:**
    - Inference time should be drastically reduced for networks with many equivalent parameters

- **Procedure**:
    - Use Random Matrix Multiplication to do multiplication
    - **y1 = x * W1**
    - Computation: 784 * 120 ===> 120 * num_samples



<u>Neural Network</u>

*Input: **x***

*Layer 1 output: **y1***

*Layer 1 Weight: **W1***

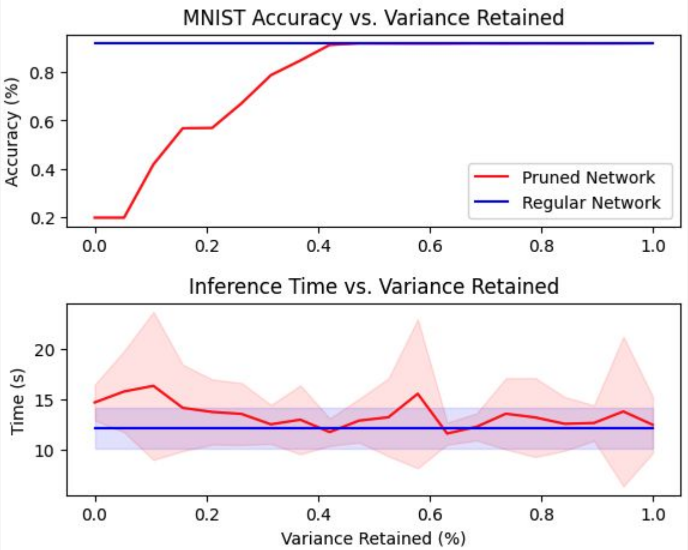
*Layer 2 Weight: **W2***

y1 = x * W1

Preliminary Results

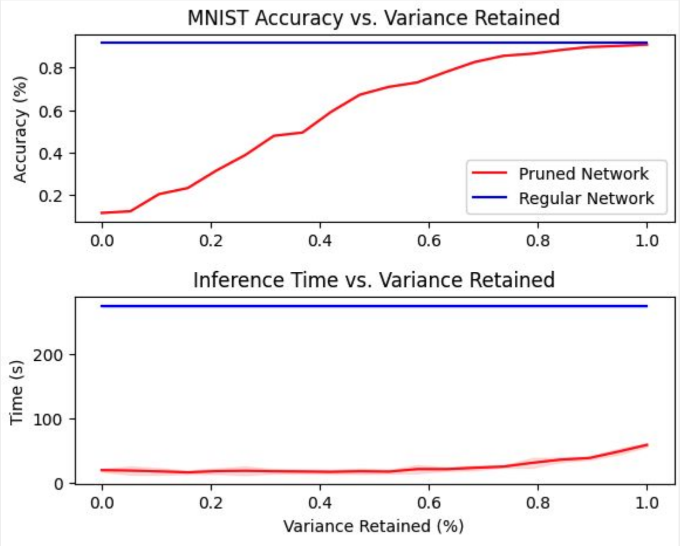
PCA:

Small Linear Network



Hidden Units: 120, 84

Large Linear Network:

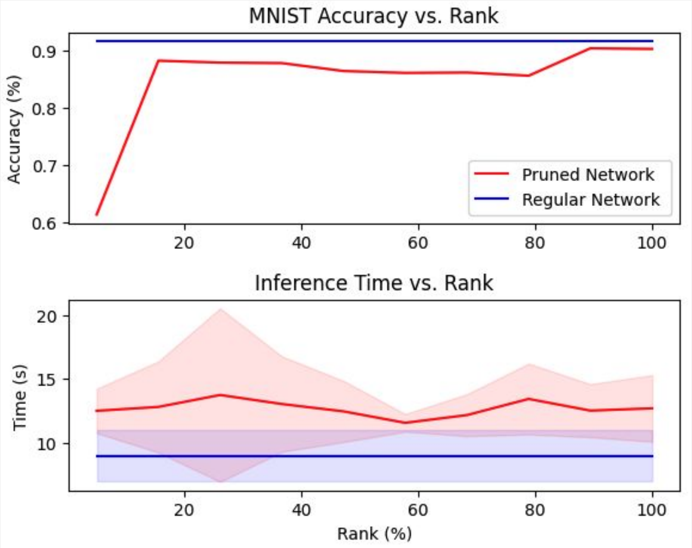


Hidden Units: 128, 512, 2048, 8192, 2048, 512, 128

Preliminary Results

Low-Rank Approximation:

Small Linear Network



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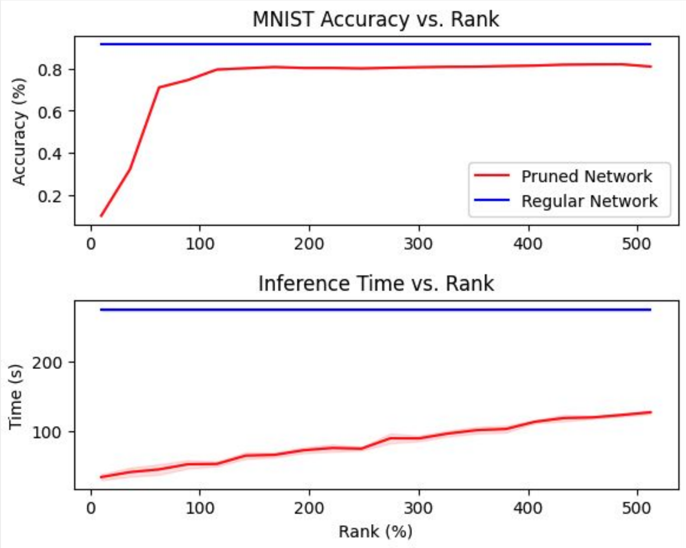
Large Linear Network:



Hidden Units: 128, 512, 2048, 8192, 2048, 512, 128

References

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4. Levin, Asriel, Todd Leen, and John Moody. "Fast pruning using principal components." Advances in neural information processing systems 6 (1993).